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Heat Transfer in Fibrous Insulations: Comparison of Theory and Experiment

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The validity of an improved theoretical model for radiative heat transfer through high-porosity fiber insulation is examined by comparison of experimental measurements with theoretical predictions of heat transfer. Radiative thermal conductivity of an optically thick medium is modeled by a diffusion approximation in which the spectral extinction properties are calculated by utilizing a rigorous treatment of the fiber medium scattering phase function and the composition of the fiber material. A semiempirical model is used to calculate the fiber-matrix conduction. The accuracy of the model is tested by comparison with experimental heat transfer data measured in vacuum for temperatures from 400 to 1500 K for three types of bonded silica fiber thermal insulation materials having different fiber size distributions. The validity of the modeling approach is demonstrated by the excellent agreement between the theoretical predictions and the experimental results.

Nomenclature

a, b = coefficients in solid thermal conductivity, Eq. (9)
B_λ = backscatter parameter
C_0 = coefficient, Eq. (12)
$e_{b\lambda}$ = Planck function
F_{res} = fiber material-dependent constant defining fiber volume fraction and geometry
f_v = fiber volume fraction
G_λ = defined by Eq. (4)
I_b = blackbody radiation
$I_{b\lambda}$ = Planck intensity function
i_λ = fiber-scattering intensity distribution
K_λ = spectral extinction coefficient
k_c = thermal conductivity of fiber matrix
k_r = radiative conductivity
k_s = thermal conductivity of fiber material
L = thickness of fiber material
N = number of fiber sizes
n = medium refractive index
p_λ = fiber-scattering phase function
q_r = radiative heat flux
q_t = total heat flux
r = radius of fiber
T = temperature
x_i = fractional volume of fibers of radius r_i
y = distance from boundary
ε = emittance
λ = wavelength
μ = direction cosine of polar angle ξ
$\sigma_{s\lambda}$ = scattering coefficient
τ_0 = optical depth
ϕ = angle of incidence on fiber
ω = azimuthal angle

Introduction

FIBROUS materials are very effective thermal insulations in high-temperature applications because of the ability of the fibers to suppress radiative energy transfer by absorption and scattering and to reduce conduction by the tortuous paths through the fiber matrix. Although at moderate temperatures the ratio of radiant-to-total heat transfer is strongly dependent upon the insulation morphology and the operating environment, such as in a vacuum or a gas, radiation is typically the major mode of heat transfer in high-porosity insulations at temperatures in excess of 400–500 K.[1] While the selection of the type of fiber insulation for a specific application depends on the temperature and environment, as well as the weight and structural requirements, the accurate understanding of the radiation transport process is essential to the optimization of thermal design. This applies to both high-porosity lightweight materials that contain loosely packed and randomly oriented fibers as well as high-density media that contain closely spaced aligned fibers. The different magnitudes of the fiber separation to diameter and wavelength ratios for the high-porosity and high-density media lead to different scattering phenomena that must be correctly accounted for in the analysis of radiation heat transfer.

Radiative transfer through fibrous media has been a subject of considerable interest for many years. Earlier studies[2–9] generally addressed radiative heat transfer by a semiempirical approach. Radiation analyses were based on approximate models using empirical parameters, most commonly an effective thermal conductivity, by fitting heat transfer data. The applicability of these models is very limited because experiments must be performed on each type of fibrous material to determine the values of the empirical parameters. Later studies[10–16] used the scattering theory for isolated infinite cylinders for the calculation of single-fiber radiative properties. Either the two-flux model utilizing a backscatter parameter or the diffusion approximation based on the Planck or Rosseland mean coefficient was then applied to calculate radiative heat transfer. The influence of material morphology, such as fiber orientation and the exact scattering phase function, on the radiative properties of the entire medium was generally not rigorously accounted for in these models. Significant discrepancies exist between measured data and predictions from these models.

Because fibers in most thermal insulations are several millimeters long and a few micrometers in diameter, they behave as two-dimensional scatterers in the thermal radiation regime. The absorption and scattering characteristics of fibrous insulations are then greatly influenced by the orientation of fibers. Radiation models that rigorously account for fiber morphology have been developed by Lee,[17–20] and they show that the extinction and scattering coefficients and scattering phase function are in general a function of the incident direction and fiber

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orientation. The orientation effect is strongest if the fibers are aligned in preferential directions. For randomly oriented fibers, the radiative properties become independent of the incident direction. These radiation models have been applied to analyze radiative transfer in fibrous media.[21–24] Lee's theoretical models were recently validated by experimental studies on the transmission and reflection characteristics of different types of high-porosity fibrous insulations.[25,26] It should be emphasized that these radiation models for fibers required only deterministic properties of the material composition and do not involve empirically adjusted factors inherent in many inversion approaches.[27]

In this work, Lee's radiation properties formulations are applied in the development of a heat transfer model for fibrous media containing randomly oriented bonded fibers. Predictions of thermal conductivity using this model are compared with a comprehensive set of heat transfer data for silica-fiber, thermal-insulation materials. The experimental data used for the comparison were selected from the literature[9] because the fiber-size distribution, solid volume fraction, and fiber optical properties, as well as the thermal performance of these materials have been well characterized. Benchmarking a heat transfer model against these independently measured data represents a rigorous validation of its theoretical basis. In this paper predictions from other previously reported models are also compared with the experimental data to evaluate the validity of the various heat transfer models for fibrous media.

Theory

The thermal insulation selected for this study is that used as the Space Shuttle Orbiter thermal protection tiles. The insulation material is fabricated using a high-temperature process, and the fibers are bonded by sintering to form a high-porosity, structurally rigid matrix.[9] The scanning electron micrograph (SEM) seen in Fig. 1 shows the size distribution, orientation, and geometric arrangement of the fibers and the sintered fiber-to-fiber bonds. Although the focus of this study is the radiative heat transfer problem, a direct correlation with the total heat transfer experimental data from Ref. 9 requires that the model for heat transfer includes the radiative component as well as that resulting from conduction within the fiber matrix. Selection of the specific experimental data to be used for comparison with prediction from the theory was based upon the high degree of experimental accuracy and the completeness of the materials characterization.

Radiative Heat Transfer

The optical depth of fibrous insulations such as the material of this study is generally of the order 10^2 for a 1-cm-thick specimen because of the very large extinction coefficient of the fibers. The fiber matrix can, therefore, be treated as an optically thick medium for radiation heat transfer. The present radiation model involves the formulation of a radiative thermal conductivity based on a modified diffusion approximation that accounts for scattering by the fiber medium. The formulation of radiative conductivity is based on the fiber extinction and scattering properties and the product of the scattering coefficient and the scattering phase function. The classic diffusion approximation has the inherent assumption that radiative energy transfer can be treated as a diffusion process in a purely absorbing optically thick medium, and it is described in many textbooks on radiation such as in Refs. 28 and 29. A number of investigators have assumed that radiative energy transfer may also be treated as a diffusion process in an absorbing and scattering medium.[16,24,30] Radiation heat transfer is then given by an equation similar to heat conduction as

$$q_r = -k_r \frac{\mathrm{d}T}{\mathrm{d}y} \tag{1}$$

where k_r is generally calculated as

$$k_r = \frac{16n^2\sigma T^3}{3} \int_0^\infty \frac{1}{K_\lambda} \frac{\mathrm{d}I_{b\lambda}}{\mathrm{d}I_b(T)}\, \mathrm{d}\lambda \tag{2}$$

The refractive index n is nearly equal to unity for high-porosity silica fiber insulations in vacuum or air.[31] The inverse of the integral over wavelength is commonly known as the Rosseland mean absorption coefficient.

In the present model scattering by fibers in a fibrous medium is accounted for by utilizing a modified extinction coefficient $\tilde{K}_\lambda$. The modified extinction coefficient is defined as

$$\tilde{K}_\lambda = K_\lambda(1 - G_\lambda) \tag{3}$$

where G_λ and K_λ are based on the models of Lee for fiber radiative properties.[17–20] The parameter G_λ is the fraction of scattered radiation resulting from radiation traversing in the forward direction. It is computed from the product of the scattering coefficient and phase function $\langle \sigma_{s\lambda} p_\lambda \rangle$ of the medium as

$$G_\lambda = \frac{1}{K_\lambda} \int_0^1 \int_{-1}^1 \langle \sigma_{s\lambda} p_\lambda(\mu, \mu') \rangle \mu'\, \mathrm{d}\mu'\, \mathrm{d}\mu \tag{4}$$

where

$$\langle \sigma_{s\lambda} p_\lambda(\mu, \mu') \rangle = \frac{2 f_v \lambda}{\pi^4} \sum_{i=1}^N \frac{x_i}{r_i^2} \int_0^{2\pi} \int_0^{\pi/2} \times \frac{i_\lambda(\eta, \phi, r_i)}{\sqrt{(1 - \cos\eta)(1 + \cos\eta - 2\sin^2\phi)}} \cos\phi\, \mathrm{d}\phi\, \mathrm{d}\omega \tag{5}$$

In Eq. (5) i_λ is the theoretical single-fiber dimensionless scattering intensity function, and η is the scattering angle given by

$$\cos\eta = \mu\mu' + (1 - \mu^2)^{1/2}(1 - \mu'^2)^{1/2} \cos(\omega - \omega') \tag{6}$$

where $\mu = \cos\xi$ is the direction cosine, and ξ is the polar angle. The extinction coefficient of a polydispersion of randomly oriented fibers is given by

$$K_\lambda = \frac{2 f_v}{\pi} \sum_{i=1}^N \frac{x_i}{r_i} \int_0^{\pi/2} Q_{e\lambda}(\phi, r_i) \cos\phi\, \mathrm{d}\phi \tag{7}$$

where $Q_{e\lambda}$ is the theoretical extinction efficiency of an infinite cylinder of radius r_i. The expressions for $Q_{e\lambda}$ and i_λ are well summarized in texts such as Kerker.[32] The subscript λ asso-

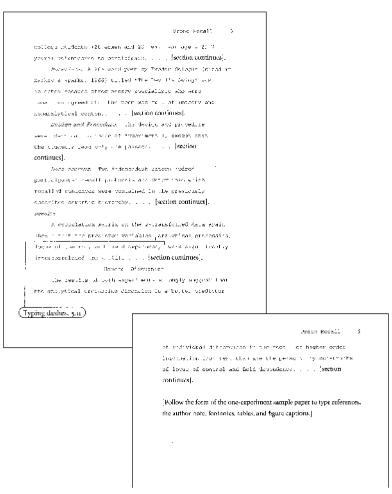

Fig. 1 SEM of type 1.7B fiber material for 2000× magnification.

ciated with the radiative properties in the preceding equations has been retained to emphasize their spectral dependence.

Conduction Heat Transfer

For the case of a rigidized-fiber thermal insulation, conduction heat transfer along the fibers and through the fiber-to-fiber fusion-bonded contacts is a significant fraction of the total heat transfer through the insulation. It has been estimated[9] that conduction is of the order of 60% of the total heat transfer under vacuum conditions at a temperature of 500 K and, depending upon fiber size, decreases to 20–35% of the total at 1300 K. A rigorous theoretical treatment of the conduction process does not appear to be feasible because of the uncertainties associated with determining valid geometric properties such as fiber-path lengths, fiber-crossing angles, effective fiber-to-fiber contact cross-sectional areas, and the complex arrangement of the series-parallel conduction paths. However, a quantitative evaluation of heat transfer by fiber conduction was done using a semiempirical model based on a unit cell and the electrical resistance network analogy. Assumptions are that the thermal conductivities of the fibers and the contact regions are the same, and the number of unit cells scales directly with insulation thickness. An equivalent resistance may be derived from experimental heat transfer data for a slab of insulation under test conditions, where conduction is the only significant transport mechanism. This was done for each insulation using the total heat transfer data measured in vacuum at a mean temperature of 115–120 K with a temperature difference of 40–50 K across a 1.25-cm-thick specimen. Calculation of the radiation transport under these test conditions shows that radiation is less than 2% of the total heat transfer.

Using the thermal conductivity of fused silica at 117 K, a factor F_{res} relating the microscale geometric properties with bulk dimensions is derived. The temperature-dependent thermal conductivity $k_s(T)$ from the literature[33] for bulk-fused silica is then combined with the geometric factor to yield an expression for the effective fiber-matrix thermal conductivity $k_c(T)$ as

$$k_c(T) = F_{res} k_s(T) \tag{8}$$

The temperature-dependent thermal conductivity data[33] for bulk silica between temperatures of 200 and 1500 K may be accurately represented by a polynomial. The temperature-dependent solid conductivity is given by

$$k_s(T) = aT^{1/3} + bT^3 \tag{9}$$

Constants a and b, determined by a least-square fit of the thermal conductivity data reported in the literature, are 0.1875 and 1.1186e-9, respectively, for k_c in units of W/m-K.

Total Heat Transfer Model

The total heat transfer through a fiber matrix includes the contributions from conduction and radiation. By utilizing the radiative conductivity based on the modified diffusion approximation, the energy equation for heat transfer by combined conduction and radiation through an optically thick planar slab is

$$\frac{d}{dy}\left[-(k_r + k_c)\frac{dT}{dy}\right] = 0 \tag{10}$$

where k_r is based on Eqs. (2) and (3) and k_c is based on Eq. (9). The total heat flux through a fiber-matrix of thickness L between two boundaries with emittance ε_1 and ε_2 at temperatures T_1 and T_2 is[29]

$$q_t = \frac{1}{C_0}\left\{\sigma(T_1^4 - T_2^4) + \frac{3k_c\tau_0}{4L}(T_1 - T_2)\right\} \tag{11}$$

where

$$C_0 = \frac{3\tau_0}{4} + \left(\frac{1}{\varepsilon_1} - \frac{1}{2}\right)\left[\frac{1}{1 + k_c/k_r(T_1)}\right] + \left(\frac{1}{\varepsilon_2} - \frac{1}{2}\right)\left[\frac{1}{1 + k_c/k_r(T_2)}\right] \tag{12}$$

and the scaled extinction coefficient given by Eq. (3) is used to compute the optical depth τ_0. An effective total thermal conductivity for the optically thick condition may be defined as the product of the total heat flux given by Eq. (11) and the thickness of the medium divided by the temperature difference between the gray opaque boundaries of the fiber material slab. For the optically thick case this effective total thermal conductivity is also equal to the sum of the radiative conductivity, Eq. (2), and the matrix conductivity, Eq. (9).

Existing Radiation Models

Predictions from other radiation heat transfer models reported in the literature are also compared with the heat transfer data on the insulation materials. These comparisons provide a benchmark for assessing the adequacy of the existing theoretical approaches to calculate radiative heat transfer through fibrous media. The existing models are based on either the two-flux approach or the diffusion approximation utilizing the Rosseland mean coefficient without appropriately correcting for scattering. The two-flux models include those by Tong and Tien[11] and Lee,[18,19] and those reported by Dombrovsky[15] and Caps et al.[16] are typical of models based on the diffusion approximation.

For the two-flux approach, the radiative heat flux through a fibrous medium of thickness L between two boundaries with emittance $\varepsilon_{\lambda 1}$ and $\varepsilon_{\lambda 2}$ at temperatures T_1 and T_2 is given by

$$q_r = \int_0^\infty \frac{e_{b\lambda}(T_1) - e_{b\lambda}(T_2)}{(1/\varepsilon_{\lambda 1} + 1/\varepsilon_{\lambda 2} - 1) + (1 - \omega_\lambda + 2B_\lambda)K_\lambda L}\,d\lambda \tag{13}$$

The basic difference between the two-flux models by Tong and Tien[11] and Lee[18,19] is in the treatment of fiber orientation in the backscatter parameter B_λ. Tong and Tien assumed that the backscatter direction relative to a fiber, irrespective of its orientation, is the same as that relative to the boundaries. The formulation of Lee, on the other hand, included a rigorous determination of the backscattering directions for any combination of incident direction and fiber orientation. Application of the conventional diffusion approximation to absorbing and scattering media is usually done by simply using the extinction coefficient in place of the absorption coefficient in the calculation of the Rosseland mean coefficient. Various weighting factors such as a mean diameter to account for the size distribution, a constant asymmetry factor for scattering, or an anisotropy factor based on the phase function for an isolated fiber were sometimes used. But a rigorous account of the scattering effects in a polydispersion of oriented fibers was absent in these diffusion approaches.

Experiment

Test Specimens

The test materials were a bonded silica fiber rigid insulation that is used in the thermal protection system of the Space Shuttle Orbiter.[9] The designation of the material is LI900, and it has a nominal bulk density of 145 kg/m^3. The compositions of these materials have been well characterized by numerous extensive measurements. The fiber volume fraction is between 0.067 and 0.074 so that independent scattering may be assumed.[34,35] The materials were produced from fibers of three different size distributions and with silica fibers from two different manufacturing sources. A typical SEM photograph of these materials is given in Fig. 1. The fiber size distributions

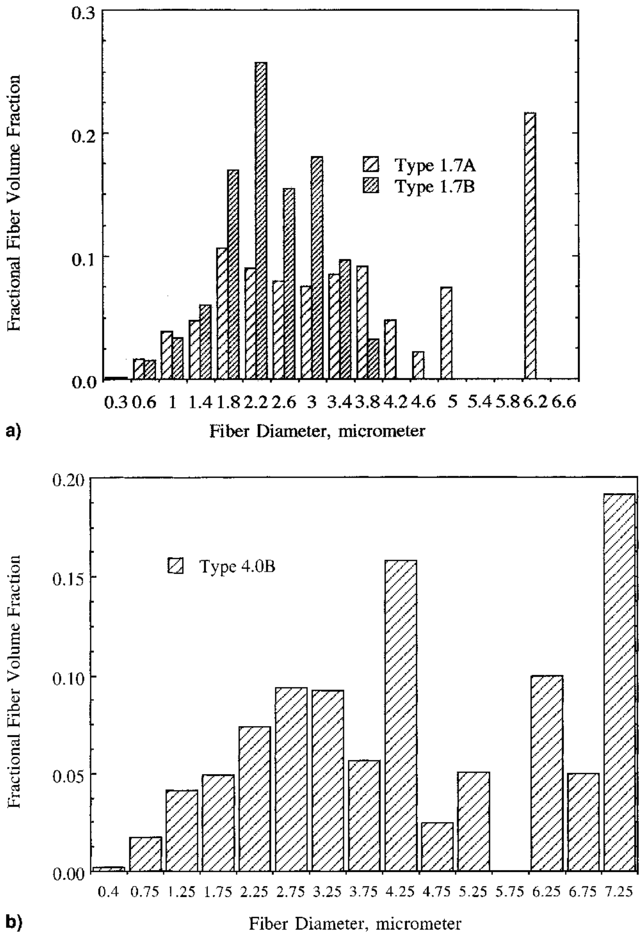

Fig. 2 Size distribution of silica fibers for a) types 1.7A and 1.7B and b) type 4.0B fiber materials.

for the three types of materials as shown in Fig. 2 were obtained from the analysis of many SEM photographs. The number of individual fibers counted in each case was typically 100. The letters A and B in the fiber-type designation denote the two sources of raw fibers which, for both cases, have a composition of >99.5% SiO_2. The mean diameters of type 1.7A and 1.7B fibers are essentially the same (1.7 denotes the mean fiber diameter in μm), but the distributions are very different. The standard deviations are 1.26 and 0.86 μm for types 1.7A and 1.7B, respectively. Type 4.0B has a mean fiber diameter of 4.0 μm and contains a large fraction of larger diameter fibers. The fiber volume fractions as determined from bulk density measurements were 0.0692, 0.0736, and 0.0677 for types 1.7A, 1.7B, and 4.0B, respectively. It is seen from Fig. 1 that the surfaces of the fibers are relatively smooth and the fiber length-to-diameter ratio is large, which is consistent with the assumption of scattering by an infinite cylinder for the theoretical model. The validity of using a spatially random fiber orientation model for these materials was verified by spectral reflectance and transmittance measurements made in orthogonal directions on the actual test materials as reported by Cunnington and Lee.[25] The type designations of A1, B2, and B2.3 in the reference correspond to 1.7A, 1.7B, and 4.0B of this work.

Thermal Conductivity Tests

The thermal conductivity data used for this study are taken from measurements performed in 1972–1974 and reported in Ref. 9. The data are based on measurements made under vacuum conditions (pressure of 1 kPa or less) using a guarded hot plate thermal conductivity apparatus. The boundary plates of the apparatus were statically oxidized Inconel 617 having a total hemispherical emittance of 0.80–0.85 over the temperature range of the testing. Test specimens were 20 cm diameter by 1.25 cm thick. Testing was conducted with a nominal 50–100 K temperature difference between the hot and cold surface plates of the specimens over the temperature range of 115–1370 K. The estimated maximum uncertainty in the thermal conductivity data is 10%. The test procedures and experimental details have been described in Ref. 9 and will not be repeated here.

Results

The physical properties required for the calculation of radiative heat transfer through the fibrous insulations are the fiber-size distribution and volume fraction, both of which are deterministic properties for each type of material. The extinction coefficient K_λ and scattering factor G_λ are then calculated according to Eqs. (4) and (7). The refractive index data for the fiber calculations are from Palik,[36] with the exception that the imaginary portion of the index data used from 2.5–6.5 μm are from Ref. 25.

The theoretical spectral values of these parameters based on Eqs. (4) and (7) for the three types of fiber materials are shown in Figs. 3 and 4. The extinction coefficient for type 1.7B is significantly greater than either type 1.7A or 4.0B in the wavelength intervals of 0.5–4.0 and 9–11 μm. The larger extinction coefficient of type 1.7B at the shorter wavelengths is a result of the narrower size distribution around the mean diameter with the larger number density of fibers grouped around the size parameter for maximum scattering efficiency. The absorption index of silica is very small for wavelengths between 0.3 and 6.0 μm, and so extinction is primarily caused by scattering. At the longer wavelengths absorption efficiency becomes

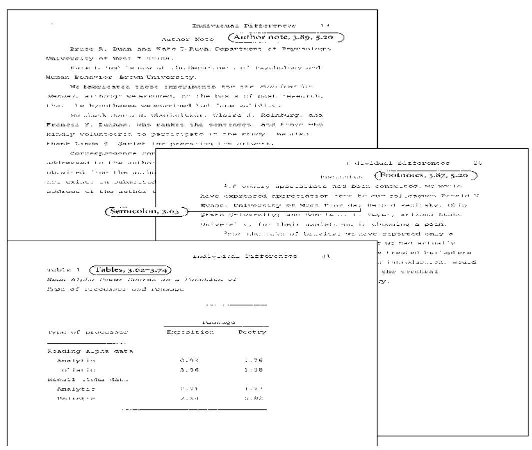

Fig. 3 Theoretical spectral extinction coefficients for types 1.7A, 1.7B, and 4.0B fiber materials.

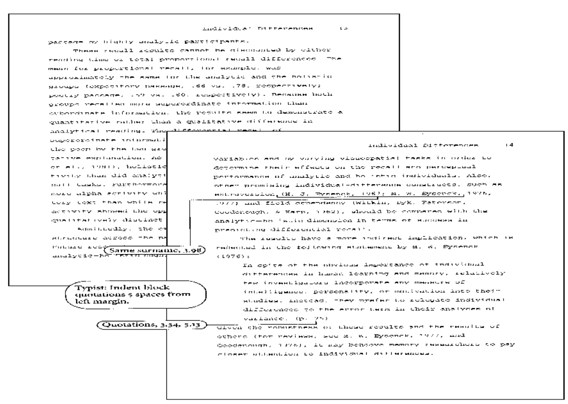

Fig. 4 Theoretical spectral variation of $(1 - G_\lambda)$ for types 1.7A, 1.7B, and 4.0B fiber materials.

important, and the type 1.7B fibers have the larger extinction coefficient. The spectral variation of G_λ is, therefore, fairly weak over the shorter wavelength region as shown in Fig. 4. The modified spectral extinction coefficient, as given by Eq. (3), is strongly influenced by fiber-size distribution because of its effect on the extinction coefficient between wavelengths from 0.5–4.0 μm, and on both K_λ and G_λ for wavelengths between 4.0 and 6.5 μm and 8 and 12 μm. The results presented in Figs. 3 and 4 show that the modified extinction coefficient is highest for type 1.7B and lowest for type 4.0B. Because the radiative conductivity is inversely proportional to this coefficient, the insulation capacity of the three fiber materials should rank in the descending order of types 1.7B, 1.7A, and 4.0B. This observation is in agreement with the experimental data for these materials, which are discussed next.

As the experimental data are based on measurements of total heat flux, the solid or matrix conduction portion of the heat flux must be first isolated from the total value to compare the predicted and measured radiative heat transfer at a given temperature. This is done by first calculating the solid conductivity using Eq. (9). The values of thermal conductivity measured at 117 K for each material that was used in conjunction with the appropriate fiber volume fractions to calculate each F_{res} value are type 1.7A, 5.30e-3 W/m-K; type 1.7B, 5.61e-3 W/m-K; and type 4.0B, 5.20e-3 W/m-K. The fused silica bulk material thermal conductivity is 0.81 W/m-K at 117 K. The resultant solid thermal conductivity values for the three materials are shown in Fig. 5, and the equations describing the temperature dependent functions are shown in the figure.

The radiative and conduction conductivities are calculated as a function of temperature, as shown in Fig. 6. The calculated total thermal conductivity is equal to the sum of the conduction and radiation conductivities at each temperature. As a consistency check, the total thermal conductivity values were also calculated from the heat-flux formulations of Eqs. (11) and (12). All of these computations were performed using a boundary temperature difference of 100 K to be consistent with the experimental conditions and thicknesses of each test specimen. The results based upon heat flux agree with those from the simple addition to within 0.5% in all cases, which is expected for the optically thick condition. For the three fiber types, solid conduction is the dominant heat transfer mode at temperatures below 400 K. Conduction and radiation are of nearly equal magnitudes at 600 K, and above 1200 K heat transfer by radiation becomes two to three times greater than that by conduction.

Figure 7 shows the comparison between the experimental data and the computed results from the present heat transfer model based on the radiative properties of Eqs. (3–6). The vertical bars on the data points represent the maximum uncertainty in a measured value.[9] A comprehensive analysis of the possible maximum uncertainty associated with the numerical values was not performed for this study. A conservative estimate of the uncertainty of the calculated values based upon uncertainties in fiber optical and physical properties is 7% at 400 K, and increases to 10% at 1500 K. The increasing uncertainty with temperature is the result of the uncertainty in the temperature dependence of the refractive index of silica. The agreement between experiment and theory is generally very good as the data fall well within the composite uncertainty for experiment and computation. Even assuming no uncertainty for the computational results, all values, with one exception, fall within the experimental data error band. However, there does appear to be a small discrepancy between theory and experiment in the temperature dependence of the experimental data at the highest temperatures. The greatest difference is seen for the type 4.0B material that has the highest radiative thermal conductivity. A similar trend, but to a lesser extent, is also seen for type 1.7A material. One possible cause for this difference may lie in the complex refractive index data used for the calculations. The data available in the literature for the real part of the refractive index are for material at essentially room temperature. Because of the absence of data on the effect of temperature on refractive index, the temperature dependence of optical properties was not included in the analysis. Because fiber extinction is primarily because of scattering at short wavelengths, a small decrease in the refractive index n at higher temperatures would decrease the scattering efficiency and increase radiation heat transfer.

Comparisons of the experimental data from Fig. 7 with calculated total thermal conductivities using the fiber-matrix conductivites of Fig. 5 and radiative conductivities derived from existing two-flux and diffusion models are shown in Fig. 8. The boundary-surface emittances used in the calculations are assumed to be 0.8 to be consistent with the experimental parameters. Because of the very large optical thickness of the test specimens, varying emittance between 0.80 and 1.0 results

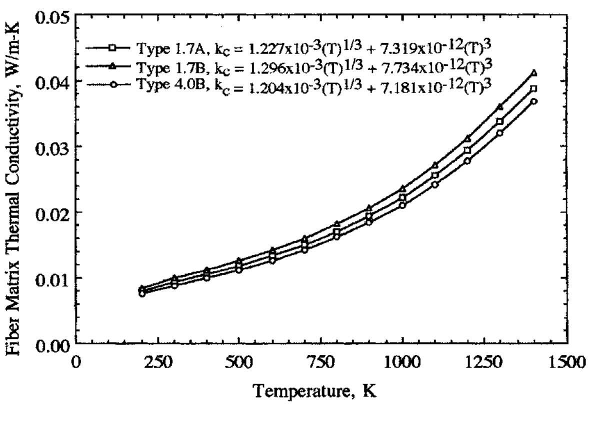

Fig. 5 Calculated fiber matrix thermal conductivity for types 1.7A, 1.7B, and 4.0B fiber materials.

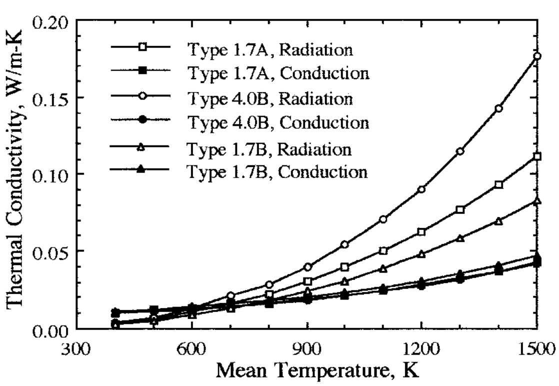

Fig. 6 Calculated radiation and fiber matrix conduction thermal conductivities for the three types of fiber materials.

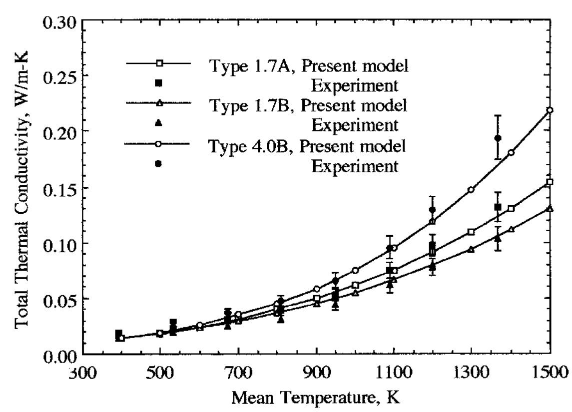

Fig. 7 Comparison between the predicted and experimental total thermal conductivities.

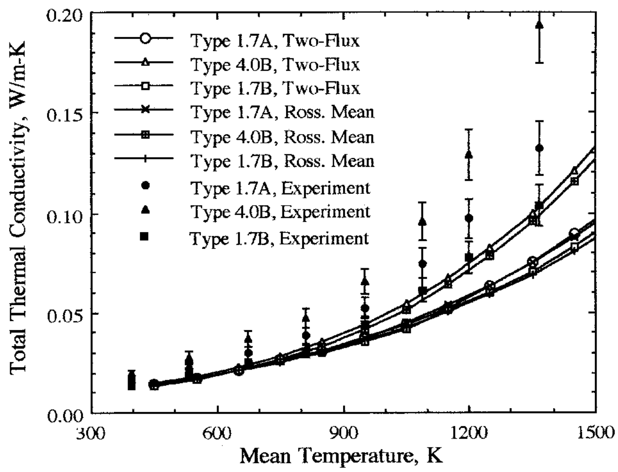

Fig. 8 Comparison of experimental data with the predicted total thermal conductivities based on the two-flux model and the conventional Rosseland mean coefficient.

in less than a 0.10% difference in the calculated value of thermal conductivity. Despite the rigorous accounting of fiber orientation in Lee's formulation[18,19] of the backscatter parameter, Lee's two-flux model predictions are only about 5–10% higher than those by Tong and Tien's model.[11] Therefore, the two-flux predictions shown in Fig. 8 refer to both models to avoid cluttering the figure. The results designated as Rosseland Mean refers to those based on the extinction coefficient as given by Eq. (2). The very poor agreement of the two-flux and Rosseland mean results with experimental data revealed the deficiency of these modeling methodologies for radiative heat transfer in fibrous media. Comparing the results of Fig. 7 with those of Fig. 8 shows the significant improvement in accuracy of the present model compared to the two-flux and Rosseland mean approaches.

Conclusions

The excellent agreement between experimental data and the results of the present heat transfer model demonstrates the validity of this theoretical approach. The present radiation model is based on a modified diffusion approximation which applies Lee's theoretical radiative properties[17–20] in the formulation of the radiative conductivity. The pertinent aspect of this formulation is the application of the scattering factor G_λ based on the rigorous scattering phase function for fibrous media. The present radiation model does not involve any adjustable constants and uses only deterministic values of the material composition. The fiber conduction model uses a geometry factor which is derived empirically from measurements of thermal conductivity at cryogenic temperature for each fiber material. Numerical and experimental results fall well within the error band associated with the composite data. With one exception, all 24 experimental data points fall within 7% of the nominal theoretical values over the entire temperature range of measurements from 400 to 1367 K. The conventional Rosseland mean coefficient diffusion model and two-flux models are shown to significantly underpredict radiation heat transfer because they do not accurately account for the scattering effects as does the rigorously derived scattering phase function.

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